A simple THz Frequency Meter

1. Introduction

The electromagnetic wave in the frequency range from 0.1 to 10 terahertz (THz) are attracting more and more attentions due to its unique property that some materials have characteristic "fingerprints" in THz range. There are two kinds of THz wave sources: broadband source and narrowband sources. Broadband source based on photoconductivity effect can provide the THz wave with a bandwidth up to several THz. And its spectrum can be measured using THz time-domain spectroscopy (TDS). However, the wavelengths (frequencies) of THz narrowband sources, such as quantum cascade laser (QCL) and difference-frequency generation of THz wave, are difficult to be measured since there is no THz spectrometer. Usually, people have to use Fourier transform infrared spectroscopy (FTIR) to measure such the wavelength information. However FTIR is very expensive and hard to use.

In this technology disclosure, we propose a novel and simple wavelength meter for narrowband THz source measurement. This technology is based on the detection of polarization state when the THz wave passes through a birefringent crystal. Then the polarization information can be converted into the wavelength information.

2. Polarization state evolution of THz wave passing through a birefringent material

Birefringence is defined as the refractive index difference between the ordinary and extraordinary rays (o-ray and e-ray) $\Delta n = n_e - n_o$. The phase difference between o-ray and e-ray after passing through a birefringent crystal should be approximately

$$\delta = 2\pi l f \Delta n / c \qquad (1)$$

Where l is the material thickness, f is the optical frequency and c is the velocity of light in vacuum. When its birefringent axis is parallel to the x-axis, Its Mueller matrix can be written as

$$\mathbf{M}_0 = T_u \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\delta & \sin\delta \\ 0 & 0 & -\sin\delta & \cos\delta \end{pmatrix} \qquad (2)$$

Where T_u is the attenuation. When its birefringent axis has a 45-degree angle with the x-axis, Its Mueller matrix can be written as

$$\mathbf{M}_{45} = T_u \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\delta & 0 & -\sin\delta \\ 0 & 0 & 1 & 0 \\ 0 & \sin\delta & 0 & \cos\delta \end{pmatrix} \tag{3}$$

From Eq. (2) and Eq. (3), Mueller matrix is a function of the frequency of THz wave. Hence, after passing through the birefringent material, the THz waves with different wavelengths will have different polarization states. If the relationship between the polarization state and the THz frequency is pre-known, the THz wavelength can be obtained by measuring the polarization state. This is the basic idea of our technology.

3. Polarimetric THz Wavelength Meter

The configuration of the proposed THz wavelength meter is shown in Fig. 1. Both THz polarizers are fixed at the angle of o degree. The first one is used to make the polarization state of input THz wave as $\vec{S}_{in} = (1, 1, 0, 0)^T$ (T denotes the matrix transpose). The second polarizer is used to convert the polarization changing to the power deviation. Their Mueller matrices are

$$\mathbf{M}_P = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{4}$$

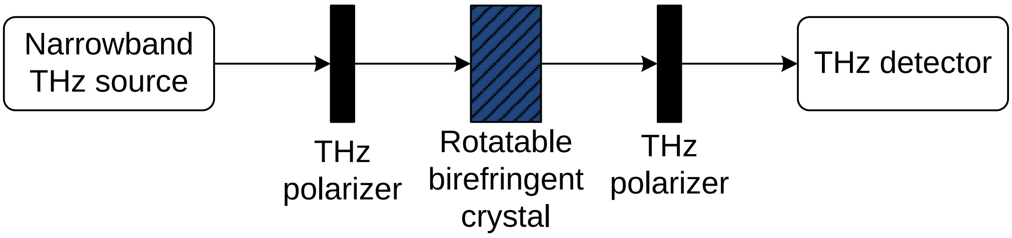


Fig.1. The configuration of the THz wavelength meter.

The output polarization state after the second THz polarizer should be

$$\vec{S}_{out} = \mathbf{M}\mathbf{M}_p \vec{S}_{in} \tag{5}$$

Firstly, we rotate the birefringent crystal to the angle of 0 degree, then based on Eq. (2), we have

$$\vec{S}_{out} = T_u (1, 1, 0, 0)^T \tag{6}$$

Secondly, we rotate the birefringent crystal to the angle of 45 degree, then based on Eq. (3), we have

$$\vec{S}_{out} = \frac{T_u}{2}(1+\cos\delta, 1+\cos\delta, 0, 0)^T \tag{7}$$

Since the usually-used THz detector, such as a bolometer, only responses to the power of the THz wave, only the first element of the Stokes vector in Eqs. (6) and (7) can be measured. But that is enough. From Eqs. (6) and (7), $\cos\delta$ can be calculated by such a measurement.

To verify the relationship between $\cos\delta$ and the frequency, we can use the THz-TDS to measure the same birefringent crystal. A measurement result of a 5mm-thick quartz crystal is shown in Fig. 2 [4].

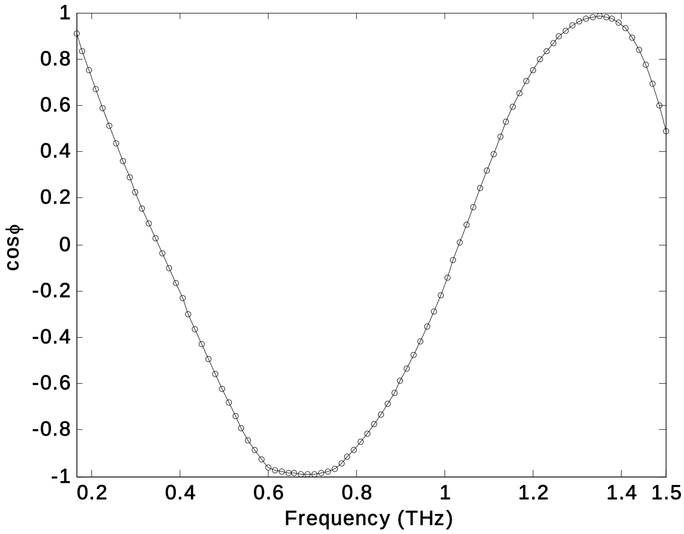


Fig. 2.The relationship between $\cos\delta$ *and the THz frequency of a quartz crystal.*

4. Measurement result

To verify this technology, a 300 GHz THz narrowband source are tested using the setup shown in Fig. 1. The structure of this THz source is illustrated in Fig. 3.

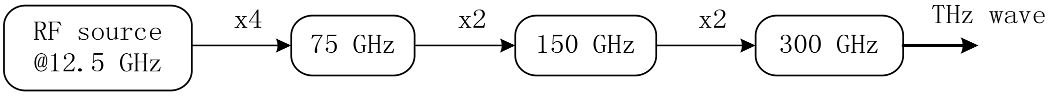


Fig. 3.The 300G Hz THz source used in the measurement.

In the experiment, a 3 mm quartz crystal and a 6 mm quartz crystal are used as the birefringent materials. The use of two crystals is to determine the wavelength uniquely. The calibration results of two quartz crystals using THz-TDS are shown in Fig. 4.

Using the proposed technology, the frequency of the THz source under test is measured to be 299.2 GHz, which is very to close to the nominal value 300 GHz.

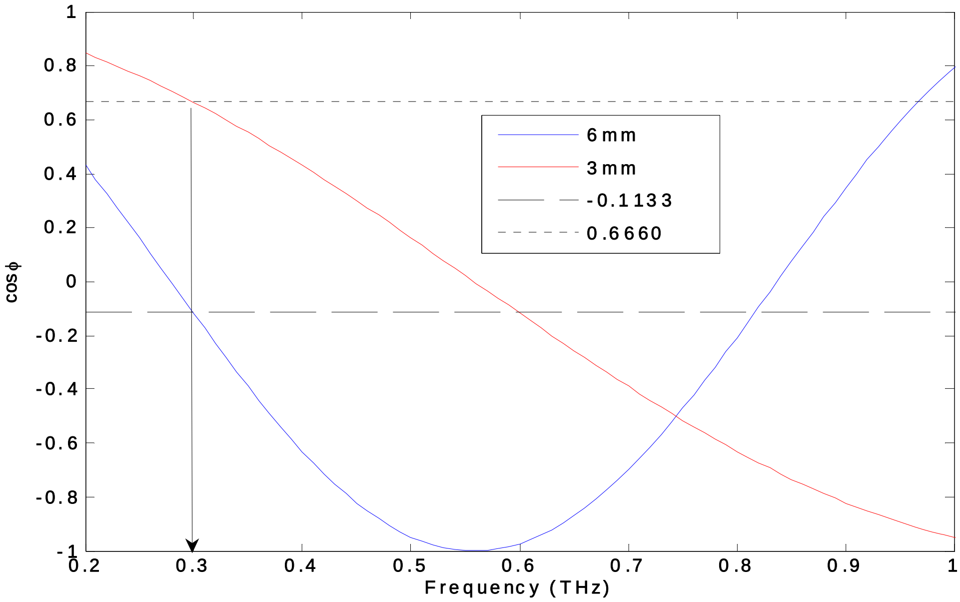


Fig. 4.Calibration curves of 3mm (red line) and 6mm (blue line) quartz crystals and the measurement results of the frequency of the THz source (arrow).

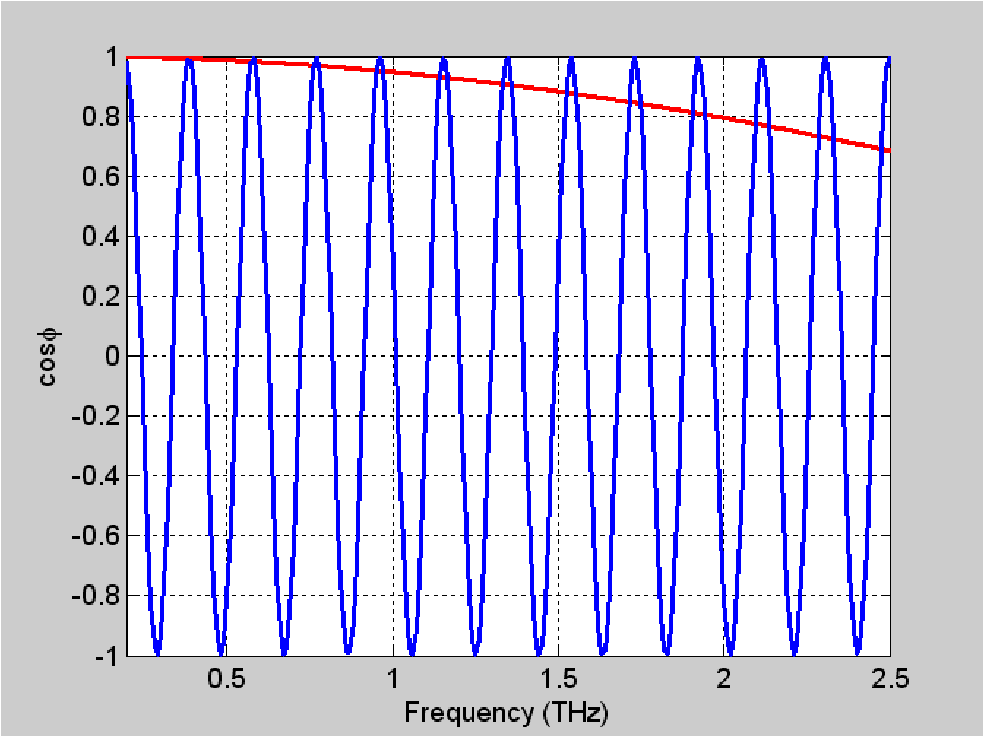
Measurement procedure

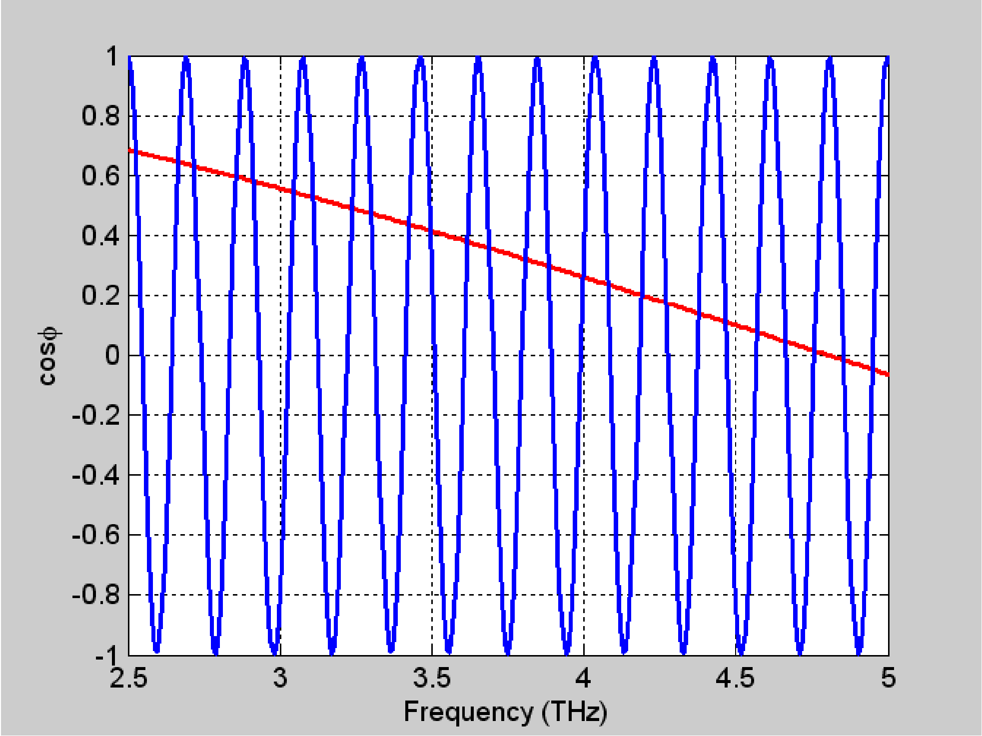
To uniquely determine the frequency in the whole THz range from 0.1 to 10 THz. We at least need two quartz crystals with different thickness. The first quartz crystal should be thin enough to guarantee the monotone in the whole THz range from 0.1 to 10 THz. For the quartz crystal we use, the thickness of the first quartz crystal should be 0.35 mm. The second quartz crystal is used to improve the measurement accuracy. The second one is chosen as 35 mm in thickness, 100 times larger than the first one. This also means 100 times more accurate than the first measurement.

The measurement procedure is summarized as follows:

1) Insert the quartz crystal with 0.35 mm thickness to the setup shown in Fig. 1.
2) Rotate it to 0 degree, record the power measured by the THz detector P_0.
3) Rotate it to 45 degree, record the power measured by the THz detector P_{45}.
4) Replace the quartz crystal with a 35 mm-thick one.
5) Repeat step 2 and 3
6) Compare the measured results $\cos\phi = 2\frac{P_{45}}{P_0} - 1$ with the curve calibrated using THz-TDS shown in Fig. 5. Then the unique and accurate wavelength (frequency) can be obtained.

Calibration results of two crystals





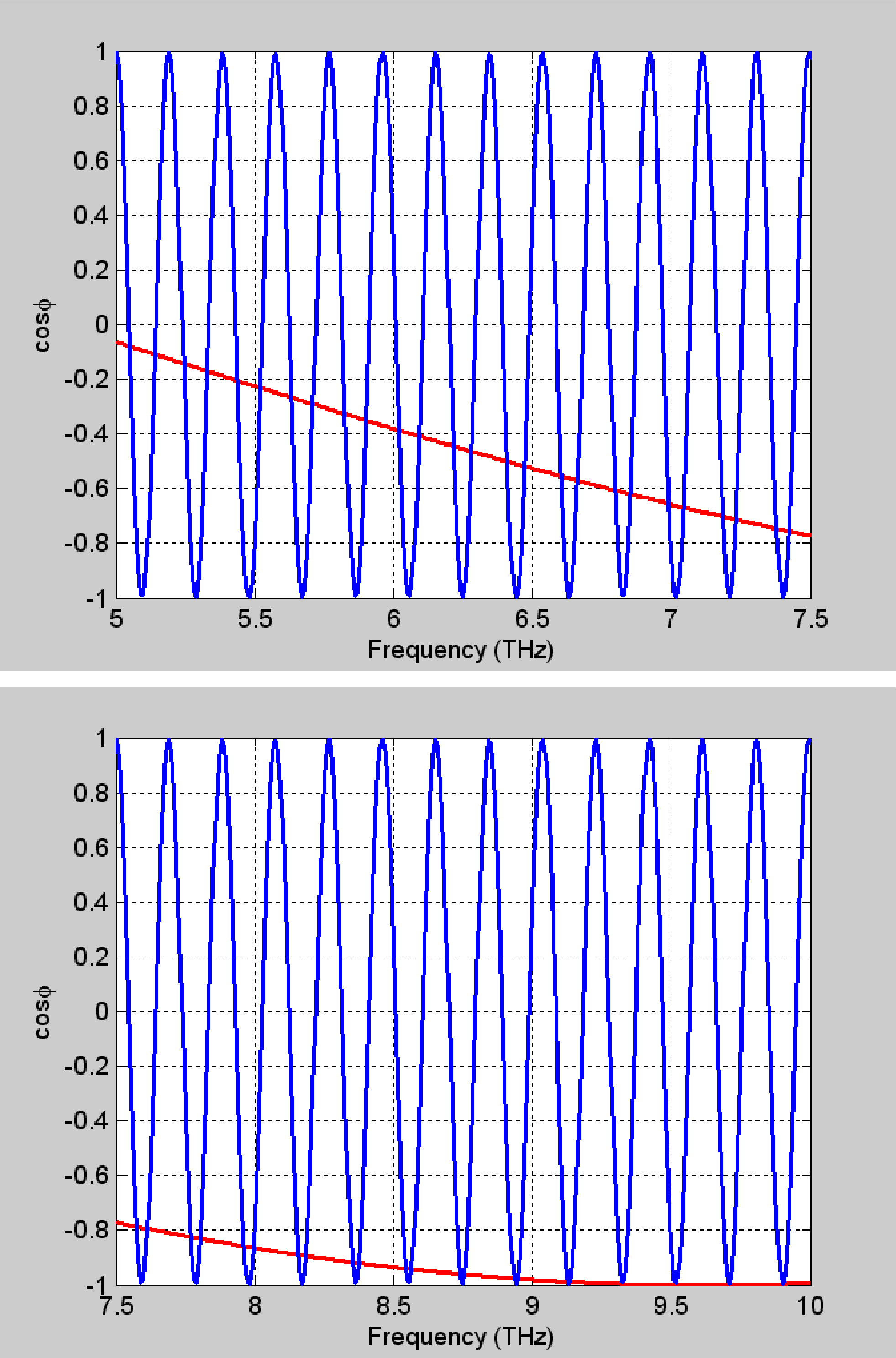


Fig. 5.The relationship between $\cos\delta$ *and the THz frequency of two quartz crystals calibrated using THz-TDS: 0.35 mm (red line), 35 mm (blue line).*